

Understanding Long-Distance Quantum Correlations

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Abstract

The interpretation of quantum mechanics (or, for that matter, of any physical theory) consists in answering the question: How can the world be for the theory to be true? That question is especially pressing in the case of the long-distance correlations predicted by Einstein, Podolsky and Rosen, and rather convincingly established during the past decades in various laboratories. I will review four different approaches to the understanding of long-distance quantum correlations: (i) the Copenhagen interpretation and some of its modern variants; (ii) Bohmian mechanics of spin-carrying particles; (iii) Cramer's transactional interpretation; and (iv) the Hess-Philipp analysis of extended parameter spaces.

KEY WORDS: Long-distance correlations; Quantum mechanics; Interpretation.

1 Introduction

In one of his thought-provoking discussions of the two-slit experiment, Feynman [1] expressed the view that “it is safe to say that no one understands quantum mechanics. [...] Nobody knows how it can be like that.” Yet 80 years of research in the foundations of the theory have led a growing number of investigators not to share Feynman's fatalism. They, in fact, have turned his assessment into a challenge, by asking “How can the world be for quantum mechanics to be true?” I have argued elsewhere [2], following others [3],

that interpreting the theory consists in providing a precise answer to this question. Moreover, I believe that providing more than one possible and consistent answer, far from introducing confusion, brings instead additional understanding, and may even stimulate the imagination.

Long-distance quantum correlations, first pointed out by Einstein, Podolsky and Rosen (EPR) [4], and sharply investigated by Bell [5], have long been considered paradoxical and in need of explanation. It is the purpose of this contribution to briefly review and analyze four different approaches through which one can make sense of them.

2 Long-distance correlations

Following Bohm [6], I consider two spin 1/2 particles prepared in the singlet state $|\chi\rangle$ and leaving in opposite directions (Fig. 1). On each side, an apparatus can measure the component of the spin of the associated particle either along axis \hat{n} or along axis \hat{n}' .

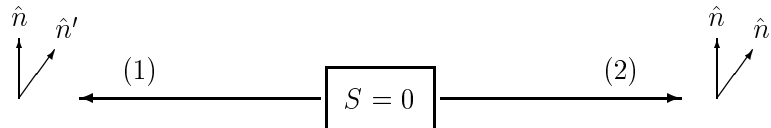


Figure 1: Two particles prepared in the singlet state and leaving in opposite directions.

In a nutshell, the paradoxical features of the arrangement can be expressed as follows:

1. Quantum mechanics predicts, and experiments confirm, that there is perfect anticorrelation when the spins of both particles are measured along the same axis.
2. This seems to suggest, against conventional quantum mechanics, that all spin components have values even before they are measured. This assertion is an instance of *local realism*.
3. Quantum mechanics predicts, and experiments confirm, that the spin correlations are in general given by

$$\langle \chi | \vec{\sigma}_1 \cdot \hat{n} \otimes \vec{\sigma}_2 \cdot \hat{n}' | \chi \rangle = -\hat{n} \cdot \hat{n}'. \quad (1)$$

4. Conventional wisdom holds that local realistic theories imply, against quantum mechanics, that the spin correlations satisfy the Bell inequalities. These are inconsistent with Eq. 1 and are experimentally violated.

3 Copenhagen and related views

In his reply to the EPR paper, Bohr [7] emphasized the holistic aspect of measurement. For him, the whole experimental setup is inseparable. The measurement of a physical quantity on one side fundamentally alters the conditions of the measurement of a conjugate variable on the other side. It prevents the definition of meaningful “elements of reality” pertaining to one part of a system only. Since no value can be ascribed to an observable outside its measurement context, the inference from statement (1) to statement (2) in the last section is, for Bohr, unwarranted.

Among the leading exponents of the Copenhagen interpretation, Heisenberg [8] has expressed the view that the state vector represents knowledge rather than the state of an independent object. The development of quantum information theory has led to renewed interest in this *epistemic view* of quantum states. To quote a recent column [9], “the time dependence of the wave function does not represent the evolution of a physical system. It only gives the evolution of our probabilities for the outcomes of potential experiments on that system.” In the epistemic view, or at least in the more radical variants of it, there are no microscopic carriers of elements of reality. The state vector is simply a device to predict correlations between distant local measurements. The correlations don’t stand in need of further explanation.

That quantum mechanics is about information is also stressed in the relational view advocated by Rovelli [10, 11]. In relational quantum mechanics, all systems (including apparatus) are quantum mechanical. An observable can have a value with respect to an observer and not with respect to another.

Specifically, spin $\vec{\sigma}_2 \cdot \hat{n}'$ has a value for observer O_1 only if O_1 measures it, or measures the result obtained by O_2 . The meaningful correlations are not those between $\vec{\sigma}_1 \cdot \hat{n}$ measured by O_1 and $\vec{\sigma}_2 \cdot \hat{n}'$ measured by O_2 , but (say) those between $\vec{\sigma}_1 \cdot \hat{n}$ and $\vec{\sigma}_2 \cdot \hat{n}'$ both measured by O_1 . Hence there is no problem with locality. The price one has to pay for this resolution of the paradox is, however, a rather significant weakening of realism. Specifically, statements like “the measurement of this observable has yielded that value” no longer hold in an absolute way.

4 Bohmian mechanics

The Schrödinger wave function for two spinless particles can be written as

$$\Psi(\vec{r}_1, \vec{r}_2, t) = \rho(\vec{r}_1, \vec{r}_2, t) \exp \left\{ \frac{i}{\hbar} S(\vec{r}_1, \vec{r}_2, t) \right\}. \quad (2)$$

In Bohmian mechanics, the particles follow deterministic trajectories governed by [12, 13]

$$\vec{v}_1 = \frac{1}{m_1} \vec{\nabla}_1 S, \quad \vec{v}_2 = \frac{1}{m_2} \vec{\nabla}_2 S. \quad (3)$$

The statistical predictions of quantum mechanics are recovered by postulating that the particles are drawn from an ensemble with probability density $|\Psi|^2 = \rho^2$.

For a factorizable wave function like

$$\Psi(\vec{r}_1, \vec{r}_2, t) = \psi_1(\vec{r}_1, t) \psi_2(\vec{r}_2, t), \quad (4)$$

we get $S(\vec{r}_1, \vec{r}_2, t) = S_1(\vec{r}_1, t) + S_2(\vec{r}_2, t)$, and the motion of particle 1 is independent of what happens to particle 2. But for an entangled wave function like

$$\Psi(\vec{r}_1, \vec{r}_2, t) = \sum_k \psi_1^{(k)}(\vec{r}_1, t) \psi_2^{(k)}(\vec{r}_2, t), \quad (5)$$

the function $S(\vec{r}_1, \vec{r}_2, t)$ does not break up into the sum of a function of \vec{r}_1 and a function of \vec{r}_2 . What happens to particle 2 instantaneously affects the motion of particle 1. From this one may be tempted to conclude that Bohmian mechanics will allow for superluminal transfer of information. This is indeed the case if state preparation is not suitably restricted [14]. But if particles are prepared with a probability density $|\Psi(\vec{r}_1, \vec{r}_2, t_0)|^2$ at time t_0 , they evolve into a density $|\Psi(\vec{r}_1, \vec{r}_2, t)|^2$ at any time t , and one can show that no superluminal transfer of information is possible.

To incorporate spin in Bohmian mechanics, one adds spinor indices to the wave function, in such a way that $\Psi \rightarrow \Psi_{i_1 i_2}$. There can be several ways to associate particle spin vectors with the wave function [12], but one way or other they involve the expressions

$$\vec{s}_1 = \frac{\hbar}{2\Psi^\dagger \Psi} \Psi^\dagger \vec{\sigma}_1 \Psi, \quad \vec{s}_2 = \frac{\hbar}{2\Psi^\dagger \Psi} \Psi^\dagger \vec{\sigma}_2 \Psi. \quad (6)$$

In the singlet state, the initial wave function typically has the form

$$\Psi = \psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\frac{1}{\sqrt{2}}(u_{1+}u_{2-} - u_{1-}u_{2+}), \quad (7)$$

in obvious notation. With such wave function, it is easy to show that $\vec{s}_1 = 0$ and $\vec{s}_2 = 0$. That is, both particles initially have spin zero. This underscores the fact that in Bohmian mechanics, values of observables outside a measurement context do not in general coincide with operator eigenvalues.

Spin measurement was analyzed in detail in Refs. [15] and [16]. In the EPR context, in particular, Dewdney, Holland and Kyprianidis first wrote down the two-particle Pauli equation adapted to the situation shown in Fig. 1. With Gaussian initial wave packets ψ_1 and ψ_2 , the equation can be solved under suitable approximations. Bohmian trajectories can then be obtained by solving Eq. 3. These involve the various components of the two-particle wave function in a rather complicated way, and must be treated numerically.

Suppose that the magnetic field in the Stern–Gerlach apparatus on the left of Fig. 1 is oriented in the \hat{n} direction. Consider the case where particle 1 enters that apparatus much before particle 2 enters the one on the right-hand side. What was shown was the following. When particle 1 enters the apparatus along a specific Bohmian trajectory, the various forces implicit in Eq. 3 affect both the trajectory and the spin vector, the latter building up through interaction with the magnetic field. The beam in which particle 1 eventually ends up depends on its initial position. If particle 1 ends up in the upper beam of the Stern–Gerlach apparatus, its spin becomes aligned with \hat{n} . Meanwhile there is an instantaneous action on particle 2, simultaneously aligning its spin in the $-\hat{n}$ direction. Similarly, if particle 1’s initial position is such that it ends up in the lower beam, its spin becomes aligned with $-\hat{n}$, and the spin of particle 2 simultaneously aligns in the \hat{n} direction.

Thus the nonlocal forces inherent in Bohmian mechanics have, once the measurement of the spin of particle 1 has been completed, resulted in particle 2 having a spin exactly opposed. It is then easy to see that if particle 2 later enters a Stern–Gerlach apparatus with magnetic field oriented in the \hat{n}' direction, its deflection in the upper or lower beam will precisely reproduce the correlations of Eq. 1.

5 The transactional interpretation

Cramer’s transactional interpretation [17, 18] is inspired by the Wheeler–Feynman electromagnetic theory, in which advanced electromagnetic waves are as important as retarded waves.

In this interpretation, a quantum process (e.g. the emission of an α particle, followed by its absorption by one of several detectors) is held to involve the exchange of offer waves (solutions of the Schrödinger equation) and confirmation waves (complex conjugates of the former). The confirmation waves propagate backward in time.

Suppose that D, at point \vec{r} , is one of a number of detectors that can absorb the particle. The offer wave, emitted at t_0 from the α particle source, will arrive at D with an amplitude proportional to $\psi(\vec{r}, t)$, the Schrödinger wave function. The confirmation wave produced by D is stimulated by the offer wave, and Cramer argues that it arrives back at the source with an amplitude proportional to $\psi(\vec{r}, t)\psi^*(\vec{r}, t) = |\psi(\vec{r}, t)|^2$. Similar offer and confirmation waves are exchanged between the source and all potential detectors, and all confirmation waves reach the source exactly at t_0 , the time of emission. Eventually, what Cramer calls a *transaction* is established between the source and one of the detectors, with a probability proportional to the amplitude of the associated confirmation wave at the source. The quantum process is then completed.

Fig. 2 is a space-time representation of an EPR setup, in the transactional interpretation. Arrows pointing in the positive time direction label offer waves, and those pointing in the negative direction label confirmation waves. Two particles are emitted by the source, and in Cramer’s sense each particle can be absorbed by two detectors, corresponding to the two beams in which each particle can emerge upon leaving its Stern–Gerlach apparatus.

Let us focus on what happens on the left-hand side. An offer wave is emitted by the source, and in going through the Stern–Gerlach apparatus it splits into two parts. One part goes into the detector labelled +, and the other goes into detector –. Each detector sends back a confirmation wave, propagating backward in time through the apparatus and reaching the source at the time of emission. A transaction is eventually established, resulting in one of the detectors registering the particle. A similar process occurs on the right-hand side, with one of the two detectors on that side eventually registering the associated particle.

If offer and confirmation waves represent causal influences of some sort,

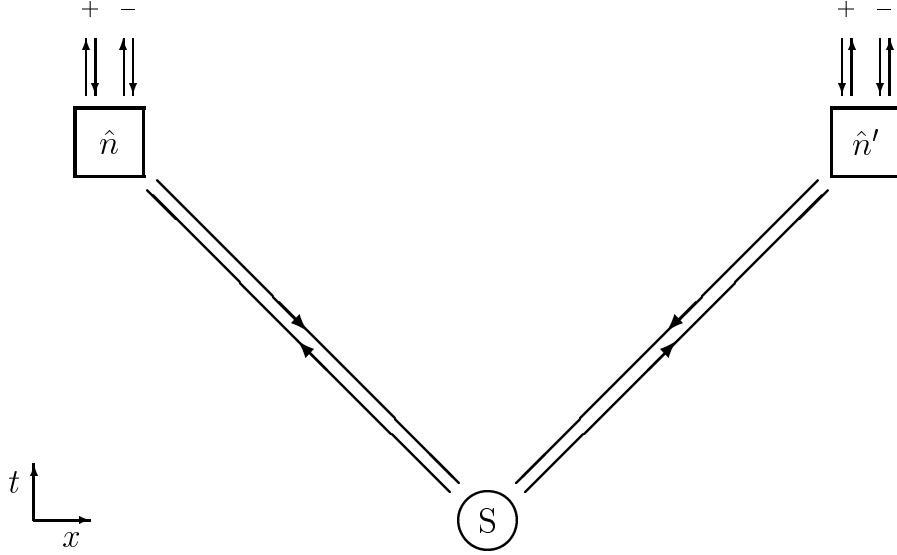


Figure 2: Offer waves (upward arrows) and confirmation waves (downward arrows) in the EPR setup.

one can see that these influences can be transmitted between the spacelike-separated detectors on different sides along paths that are entirely timelike or lightlike. In this way, the EPR correlations are explained without introducing any kind of superluminal motion.

6 Extended parameter space

Bohmian mechanics and Cramer's transactional interpretation explain the long-distance quantum correlations by means of channels which, although not allowing for the superluminal transfer of classical bits of information, involve causal links of some sort between spacelike-separated instruments. In recent work, Hess and Philipp [19] have argued that the correlations might be understood without appealing to such links.

In the original proof of his inequality, Bell [5] assumed that the state of the particle pair is characterized by a hidden variable λ , which represents one of the values of a random variable Λ . He further assumed that the result of measuring (say) the \hat{n} component of the spin of particle 1 is fully determined

by λ and \hat{n} . He supposed, however, that the result of measuring the \hat{n} component of the spin of particle 1 does not depend on which component of the spin of particle 2 is measured. The latter assumption embodies the prohibition of superluminal causal influences, and is fully endorsed by Hess and Philipp.

Hess and Philipp point out that Bell's proof, as well as all subsequent proofs of similar inequalities, make use of parameter spaces that are severely restricted. They introduce much more general spaces. Like Bell, they assume that pairs of particles emitted in the singlet state are characterized by a random variable Λ , which is stochastically independent on the settings on both sides. But then they associate with each measuring instrument random variables $\Lambda_{\hat{n}}^{(1)}(t)$ and $\Lambda_{\hat{n}'}^{(2)}(t)$, which depend both on the setting of the instrument and on the time. The result of measuring, say, the \hat{n} component of the spin of particle 1 at time t , is taken to be a deterministic function of Λ and $\Lambda_{\hat{n}}^{(1)}(t)$.

Several important remarks should be made at this stage. Firstly, and in the spirit of standard quantum mechanics, neither particle has a precise value of any of its spin components before measurement. Rather, the particles and the instruments jointly possess information that is sufficient for deterministic values to obtain upon measurement. Secondly, the dependence of the instruments' random variables on some universal time allows for a stochastic dependence of measurement results on one another, conditional on Λ , if the measurements are performed at correlated times in the two wings. And yet thirdly, the measurement result on one side can be stochastically independent on the setting on the other side.

With such extended parameter spaces, Hess and Philipp have shown that the standard proofs of the Bell inequalities come to a halt. Such proofs typically assume that the two particles, once they have left the source, simultaneously have well-defined values of more than one spin component. But in the extended parameter space approach, spin components get values only upon measurement. Counterfactual reasoning is allowed only in the sense that had a different spin component been measured, it would have yielded a definite and deterministic value. But that value does not exist before measurement. And since the measurement of different spin components requires incompatible apparatus, different spin components of the same particle cannot have values at the same time. But spin components of both particles measured at correlated times in the two wings can be stochastically dependent, through the dependence of the instrument random variables on time.

In experimental tests of the Bell inequalities, spin measurements on a given pair were performed in a time frame many orders of magnitude smaller than the time interval between successive measurements on two different pairs. It is therefore conceivable that a time dependence of the instrument random variables, having no effect on such properties as perfect anticorrelation for particles in the same run, could reproduce the quantum-mechanical long-distance correlations observed on runs performed at different times. Such runs would not sample the quantities that appear in the standard forms of the Bell inequalities.

Hess and Philipp also proposed an explicit model of Einstein-local random variables that lead to violations of the Greenberger–Horne–Zeilinger equations [20], violations that experiments claimed to have observed [21].

7 Summary and conclusion

The long-distance quantum correlations and the violation of Bell inequalities can be understood in a number of different ways, four of which were reviewed here.

In the Copenhagen and epistemic views, correlations are basically dealt with by relaxing the requirements of explanation. In Bohmian mechanics, instantaneous interactions orient the spin of the second particle while the spin of the first one is measured, but restrictions on state preparation prevent the superluminal transfer of information. In the transactional interpretation, advanced waves provide for a communication channel between spacelike-separated detectors. In the Hess–Philipp approach, finally, correlations are explained through instrument random variables that depend both on setting and on time.

I have attempted to illustrate the idea that a theory is made clearer through the display of various models that make it true. This is the process of interpretation, and in connection with it one should be wary of identifying consequences of the formalism of quantum mechanics with consequences of specific interpretations of it. This, unfortunately, has not always been done, as the example of Ref. [22] still shows.

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